Lecture 9

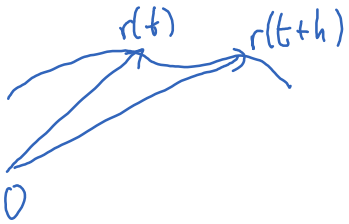
Monday, February 1, 2021 11:31 AM

* Prayer

* Spiritual thought:

* Answering questions ....

$r(t)$ ... position at time t

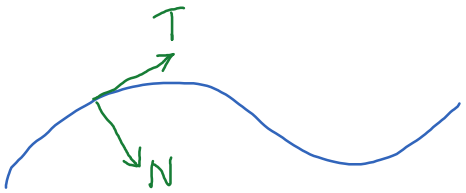


$$v(t) = r'(t) = \lim_{h \to 0} \frac{r(t+h) - r(t)}{h}$$

is the velocity.

Speed: $V(t) = |v(t)|$

Acceleration: $a(t) = r''(t)$



normal vector

We can write $a(t)$ as

$$a = \underbrace{V'(t)T}_{\text{tangential component}} + \underbrace{\kappa(t)V(t)^2 N}_{\text{normal component}}.$$

Ex:

$$r(t) = \langle t^2 - 1,\ t^3 - 4t,\ 0 \rangle$$

Find velocity, acceleration, speed, normal component of the acceleration at the position $(0, -3, 0)$

$r'(t) = \langle 2t, 3t^2 - 4, 0 \rangle$

$v(t)^2 = 4t^2 + (3t^2-4)^2 = t^4 - 4t^2 + 16 = (t^2-2)^2 + 12 \geq 12$

Minimum speed is $\sqrt{12}$, at $t = \sqrt{2}$.

$r''(t) = \langle 2, 2t, 0 \rangle$

$r'(t) \times r''(t) = \langle 0, 0, 4t^2 - 2(t^2-4) \rangle$

$= \langle 0, 0, 8 + 2t^2 \rangle$

$$\kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^{3/2}} = \frac{8 + 2t^2}{\left[(t^2-2)^2 + 12\right]^{3/2}}$$

* Problem 3 & 9, page 928.

9) The helix $r_1(t) = \langle \cos t, \sin t, t \rangle$

and the curve $r_2(t) = \langle 1+t, t^2, t^3 \rangle$

intersect at point $(1,0,0)$. Find the angle of intersection.

3) Find a vector function that represents the curve of intersection of $x^2 + y^2 = 16$ and $x + z = 16$.